DROPLET MOTION IN AN INHOMOGENEOUS-IN-TEMPERATURE VISCOUS MEDIUM

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Theory of heat carrying by spherical fluid particles in viscous liquids is developed. It is found that the velocity of the particles is directly proportional to the derivative of surface tension with respect to temperature.

In [1-4] a theory was developed of the motion of aerosol particles in temperature-inhomogeneous gases; it was found that the transfer effect of these gases is proportional to the coefficient of thermal sliding of the gas along their surface. It was assumed that the radius R of the particles greatly exceeds the length of the free path of the molecules. In [1] the case of $\lambda/R = 0$ was analyzed which corresponds to very large particles.

In later works [2, 4] a generalization was given of this theory to the case of small but finite ratios λ/R. The difference between [2] and [3, 4] lies in that the authors of [3, 4] used an improved formula for the rate of the thermal sliding of the gas on the surface of aerosol particles; this enables one to bring the theory closer to the experimental results [5, 6] obtained by using very stringent methods. However, no solution has been found so far as regards the motion of fluid spherical droplets in inhomogeneous-in-temperature viscous liquid media. The special feature of this problem consists, firstly, in that the motion of the droplets is due to temperature changes in the surface tension on the boundary of the droplet–surrounding fluid interface, and, secondly, that the droplet viscosity η_i is comparable to that of the external medium which makes us take into account the inside motion of the substance in the droplet.

It is considered that a spherical droplet of fluid of radius R is suspended in a temperature-inhomogeneous fluid. The substance of the droplet is different from that of the surrounding fluid. The gravitational forces acting on the droplet will from now on be always ignored. The droplet does not dissolve in the fluid and across its surface there is no exchange of matter with the surrounding medium. The length λ of the free path of the molecules is considerably shorter than the radius R of a particle; therefore, the Knudsen number equal to λ/R is assumed to be equal to zero. In the mathematical formulation of the problem, the origin of the spherical coordinate system (r, θ, φ) is located at the center of the droplet; moreover, it is assumed that at a great distance from the droplet the temperature gradient $(\nabla T)_\infty$ is constant and directed along the polar axis $z = r\cos\theta$.

It can be assumed as it was done when solving the Stokes problem [7, 8] that for such a choice of the origin the droplet is at rest and the center of gravity of the fluid moves with a constant velocity $\vec{u}$ at great distances from the droplet.

The computation of the rate of the carried heat is based on a prior determination of the total force $\vec{F}$ acting on the particle. To be able to evaluate the latter the distribution is required of the velocities $\vec{v}^{(e)}$ and of the pressure $p^{(e)}$ in the medium around the droplet. The velocity $\vec{v}^{(e)}$ is due to the temperature gradient $(\nabla T_e)_\infty$ and if the parameter $R(\nabla T_e)_\infty/T_{ave}$ is small compared to unity then the Reynolds number is considerably smaller than unity, as was correctly shown in [9]. The latter enables one to use the linearized Navier–Stokes equations and the continuity equation [9] to solve the problem

$$\eta_e \nabla^2 \vec{v}^{(e)} = \nabla p^{(e)}, \tag{1}$$

$$\operatorname{div} \vec{v}^{(e)} = 0. \tag{2}$$

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If the parameter $R(\nabla T_e)_\infty / T_{ave}$ is small then the Péclet number is also small (see [9]); this yields linearized heat-conduction equations for the temperature T_e outside the droplet and T_i inside it:

$$\nabla^2 T_e = 0 \quad \text{for} \quad r > R, \tag{3}$$

$$\nabla^2 T_i = 0 \quad \text{for} \quad r < R. \tag{4}$$

Linearized equations are also valid for the velocities and pressure inside the droplet, namely,

$$\eta_i \nabla^2 \vec{v}^{(i)} = \nabla p^{(i)}, \tag{5}$$

$$\operatorname{div} \vec{v}^{(i)} = 0. \tag{6}$$

For $r \to \infty$ the radial and tangential components of the velocity of the medium assume the form

$$\left.\begin{aligned} v_r^{(e)} &= |\vec{u}| \cos\theta, \\ v_\theta^{(e)} &= -|\vec{u}| \sin\theta \end{aligned}\right\} \quad \text{for} \quad r \to \infty. \tag{7}$$

The temperature T_e for $r \to \infty$ satisfies the relation

$$T_e = T_e^{(0)} + |(\nabla T_e)_\infty| r \cos\theta. \tag{8}$$

The change in the shape of the fluid droplet is ignored when viscous liquid flows past it since the distortion of the spherical shape is, as a rule, very slight (see [8]). In such a case the normal velocity components of the motion for matter outside or inside the droplet must vanish on its surface:

$$v_r^{(e)} = 0 \quad \text{for} \quad r = R, \tag{9}$$

$$v_r^{(i)} = 0 \quad \text{for} \quad r = R. \tag{10}$$

For tangential velocity components one obviously has the relation

$$v_\theta^{(e)}|_{r=R} = v_\theta^{(i)}|_{r=R}. \tag{11}$$

Since the boundary of the droplet—outside medium interface is in equilibrium, therefore the components of the tensor of total stress must be continuous on this boundary [8, 10]

$$\eta_e \left(\frac{1}{r} \frac{\partial v_r^{(e)}}{\partial \theta} + \frac{\partial v_\theta^{(e)}}{\partial r} - \frac{v_\theta^{(e)}}{r} \right)\Bigg|_{r=R} + \frac{\partial \sigma}{\partial T_e} \frac{\partial T_e}{R \partial \theta} \Bigg|_{r=R} = \eta_i \left(\frac{1}{r} \frac{\partial v_r^{(i)}}{\partial \theta} + \frac{\partial v_\theta^{(i)}}{\partial r} - \frac{v_\theta^{(i)}}{r} \right)\Bigg|_{r=R}. \tag{12}$$

The condition (12) is the condition of equality of the tangential components of the stress tensor. The boundary condition for the continuity of normal stresses is replaced by the equivalent condition of the vanishing of the total force $\vec{F}$ on the droplet in its uniform motion [8].

For $\lambda/R \to 0$ for temperatures T_e and T_i one has the following condition:

$$(T_e - T_i)|_{r=R} = 0. \tag{13}$$

Finally, the heat flux across the surface of the droplet is also continuous,

$$-\varkappa_e \frac{\partial T_e}{\partial r}\Bigg|_{r=R} = -\varkappa_i \frac{\partial T_i}{\partial r}\Bigg|_{r=R}, \tag{14}$$

where $\varkappa_e$ and $\varkappa_i$ are the heat conductivities of the outside medium and of the droplet, respectively.

Equations (1)-(6) and the boundary conditions (7)-(14) enable us to seek the solutions in the form

$$v_r^{(e)} = \left(\frac{A_e}{r^3} + \frac{B_e}{r} + |\vec{u}| \right) \cos\theta, \tag{15}$$

$$v_\theta^{(e)} = \left(\frac{A_e}{2r^3} - \frac{B_e}{2r} - |\vec{u}| \right) \sin\theta, \tag{16}$$

$$p^{(e)} = p_0^{(e)} + \eta_e \frac{B_e}{r^2} \cos\theta, \tag{17}$$

$$v_r^{(i)} = (C_i + D_i r^2) \cos\theta, \tag{18}$$

$$v_\theta^{(i)} = -(C_i + 2D_i r^2)\sin\theta, \quad (19)$$

$$p^{(i)} = p_0^{(i)} + 10\eta_i D_i r\cos\theta, \quad (20)$$

$$T_e = T_{0e} + |(\nabla T_e)|_\infty r\cos\theta + \frac{\gamma_1}{r^2}\cos\theta + \frac{\gamma_2}{r^2}, \quad (21)$$

$$T_i = \gamma_3 + \gamma_4 r\cos\theta. \quad (22)$$

Substituting the solutions (15)-(22) into the boundary conditions (9)-(14) one obtains a system of algebraic equations which enable one to determine the values of the constants A_e, B_e, C_i, D_i, γ_1, γ_2, γ_3 and γ_4. However, it is not intended here to find the analytic expressions for all the eight constants since we shall only require the analytic expression for the constant B_e.

The total force exerted on the droplet is equal to the integral over its surface of the components of the stress tensor [7, 8],

$$\vec{F} = \iint (p_{rr}^{(e)}\cos\theta - p_{r\theta}^{(e)}\sin\theta)|_{r=R}\, dS. \quad (23)$$

The integration results in

$$\vec{F} = 4\pi\eta_e\vec{B}_e, \quad (24)$$

where $\vec{B}_e$ in its vectorial form is given by

$$\vec{B}_e = \frac{(2\eta_e + 3\eta_i)}{2(\eta_e + \eta_i)} R\vec{u} - \left(\frac{\varkappa_e}{2\varkappa_e + \varkappa_i}\right) \frac{R^2 \frac{\partial\sigma}{\partial T_e}(\nabla T)_\infty}{(\eta_e + \eta_i)}. \quad (25)$$

The condition that the total force (24) must vanish indicates automatically that the vector $\vec{B}_e$ of (25) also vanishes. From the latter condition an expression is obtained for the velocity $\vec{u}$:

$$\vec{u} = \frac{2R\varkappa_e}{(2\varkappa_e + \varkappa_i)(2\eta_e + 3\eta_i)} \frac{\partial\sigma}{\partial T_e}(\nabla T)_\infty. \quad (26)$$

By changing over to the coordinate system related to the center of gravity of the surrounding medium one easily obtains from (26) the rate of heat carried by the droplet; this is done by changing the sign of $\vec{u}$:

$$\vec{u}_T = -\frac{2\varkappa_e R}{(2\varkappa_e + \varkappa_i)(2\eta_e + 3\eta_i)} \left(\frac{\partial\sigma}{\partial T_e}\right)(\nabla T)_\infty. \quad (27)$$

If the viscosity η_i of the droplet matter exceeds considerably that of the surrounding medium η_e (in the limit $\eta_i/\eta_e \to \infty$) the velocity $\vec{u}_T$ approaches zero. The velocity $\vec{u}_T$ also approaches zero if $\varkappa_i/\varkappa_e \to \infty$, that is, for very high thermal conduction of the droplet compared with that of the surrounding medium. The surface tension σ is a decreasing function of temperature. Therefore, one has $\partial\sigma/\partial T < 0$, and the rate $\vec{u}_T$ of heat carrying given by (27) is directed towards the temperature growth.

From the formula (27) the cases of motion of gas cavities in fluid are easily obtained. To this end η_i must approach zero. This yields

$$\vec{u}_T = -\frac{\varkappa_e R}{(2\varkappa_e + \varkappa_i)\eta_e} \left(\frac{\partial\sigma}{\partial T_e}\right)(\nabla T_e)_\infty. \quad (28)$$

The velocity of motion of the cavities exceeds the velocity of the liquid droplets if for both the surrounding medium is the same.

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